

# Transcritical shallow-water flow past topography: finite-amplitude theory

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## Abstract:

In this talk I will present recent results obtained jointly with Roger Grimshaw (Loughborough) and Noel Smyth (Edinburgh) on the generation of undular bores in one-dimensional fully nonlinear shallow-water flows past localised topographies [1]. The description is made in the framework of the forced Su-Gardner (a.k.a. 1D Green-Naghdi) system of equations, with a primary focus on the transcritical regime when the Froude number of the oncoming flow is close to unity. A combination of the local transcritical hydraulic solution over the localized topography, which produces upstream and downstream hydraulic jumps, and unsteady undular bore solutions describing the resolution of these hydraulic jumps, is used to describe various flow regimes depending on the combination of the topography height and the Froude number. We take advantage of the recently developed modulation theory of Su-Gardner undular bores [2] to derive the main parameters of transcritical fully nonlinear shallow-water flow, such as the leading solitary wave amplitudes for the upstream and downstream undular bores, the speeds of the undular bores edges and the drag force. Our results confirm that most of the features of the previously developed description in the framework of the uni-directional forced KdV model [3] hold up qualitatively for finite amplitude waves, while the quantitative description can be obtained in the framework of the bi-directional forced Su-Gardner system.

## References:

1. G.A. El, R. H. J. Grimshaw, and N. F. Smyth, *J. Fluid Mech.* 640, 187 - 214 (2009)
2. G.A. El, R. H. J. Grimshaw, and N. F. Smyth, *Phys. Fluids* 18, 027104 (2006).
3. R. H. J. Grimshaw and N. F. Smyth, *J. Fluid Mech.* 169, 429-464 (1986).